Analysis of the Microwave Spectrum of Tricarbon Oxide Sulfide, O=C=C=S, in Highly Excited Bending States

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In memoriam Professor Dr. Werner Zeil

The microwave spectrum of tricarbon oxide sulphide (3-thioxo-1,2-propadiene-1-one), O=C=C=C=S, has been measured in the frequency range from 8 to 40 GHz and includes vibrational satellite lines arising from the vibrational manifold of the lowest-lying doubly-degenerate bending mode v_7 . The method of analysis followed the theory of rotation-vibrational interaction as developed by Nielsen and Amat. The spectroscopic constants B_v and D_v as well as several vibration-rotational constants for each vibrational state $v_7=0$ to 7, $v_5=1$ and $v_6=1$ were determined with the aid of a newly written least squares program. The program structure relies on the correlation of the symmetry classification of energy levels in linear and bent molecules, and follows closely the analysis of asymmetric rotor spectra. From the rovibrational spectrum of C_3OS in excited states of v_7 up to $v_7=7$ and l=6 the sign and magnitude of the effective vibrational anharmonicity constant $x_{l,l,l}$ were determined. The interpretation of these results yields the information that in C_3OS the potential function describing the two-dimensional oscillator of the v_7 bending mode is very harmonic and does not contain a perturbing hump. The dynamic behaviour of C_3OS , a classical example of a linear molecule, is thus in sharp contrast to the quasilinear behaviour of C_3O_2 .

I. Introduction

The pure rotational spectrum of 3-thioxo-1,2-propadien-1-one, O=C=C=C=S, was reported recently by Winnewisser and Christiansen [1]. The microwave spectrum of this molecule is that of a typical linear molecule and shows no sign of quasilinear behaviour. This is in sharp contrast to the complex infrared and submillimeter wave spectrum [2, 3] of O=C=C=C=O, which is almost half-way between a linear and a bent molecule [4, 5].

The molecule C_3OS has three degenerate bending modes which have first been estimated from relative intensity measurements [1] and were later confirmed by the infrared observations of Nicolaisen and Christiansen [6] to be $v_5 = 529 \text{ cm}^{-1}$, $v_6 = 429 \text{ cm}^{-1}$ and $v_7 = 77 \text{ cm}^{-1}$ (relative intensity estimates)

served easily. In this paper we report our measurements and the analysis of C_3OS molecules in some of these vibrational states. A further publication will be concerned with the potential function of the ν_7 vibrational manifold as determined from relative intensity measurements of the reported vibrational satellite lines and the variation of the molecular electric dipole moment with the vibrational quantum number ν_7 [7]. A third paper in this series will be concerned with the molecular structure of C_3OS [8].

mate 83(10) cm⁻¹). Therefore the vibrational satellite lines in the pure rotational spectrum can be

observed at room temperature from molecules in a

large number of excited vibrational states. The

energy levels of the v₇ vibrational manifold are

especially highly populated at room temperature,

so that lines from states up to $v_7 = 18$ can be ob-

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II. Experimental Procedures

a) Chemical Synthesis of C₃OS

Samples of C_3OS were prepared by the reaction of C_3O_2 with P_4S_{10} as described in [1]. The experi-

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mental set-up used in the preparation is shown in Figure 1. As can be seen from Figure 1, the synthesis of C₃OS starts with liquid C₃O₂ which is kept in a storage vessel at -40 °C. Its vapours are allowed to pass over a stirred, saturated aqueous solution of CaCl₂ at 20 °C in order to pick up a defined amount of water vapour. The substitution reaction of the replacement of oxygen in C₃O₂ by sulfur takes place in a U-tube (see Fig. 1) which contains a mixture of 10 g of a coarse fraction of Celite 545, 10 g of P₄S₁₀ and 5 g of paraffin oil through which the moisturized C₃O₂ is passed. The reactor tube is heated in an oil bath to 130-135 °C. The volatile reaction products are passed through a cold-trap at -75 °C in which the less volatile fraction is collected, such as the acid components CH₃COOH etc. The volatile fraction is then trapped with liquid nitrogen. This fraction consists mainly of C₃O₂ and a small amount of C₃OS. C₃O₂ and C₃OS can easily be separated due to the difference in vapour pressure - 70°C. The C₃OS is further purified by a single-plate distillation from -80 °C into a sample tube at liquid nitrogen temperature. It is important for the purification that the contents of the C₃OS trap are repeatedly melted and recooled to dry ice temperature during the

distillation process. In this way some impurities are kept back. Unreacted C_3O_2 is fed back into a C_3O_2 storage vessel. The yield of this process is extremely low. Only about 0.2% of C_3O_2 is converted into C_3OS at each pass. In order to improve the yield a cyclic procedure has been adapted as can be seen from Figure 1. It was found that for C_3OS at -78 °C the vapour pressure is about 0.08 mbar.

b) Microwave Measurements

The microwave spectrum of C₃OS was recorded using a Hewlett-Packard spectrometer model MRR 8460 A. The frequency range covered by this spectrometer is 8 to 40 GHz. All measurements were carried out at room temperature. The reproducibility of the reported frequencies is 10 to 20 kHz. The C₃OS sample showed about 15% decrease of the line intensities after several hours in the gold-plated absorption cell at a pressure of 0.025 mbar. The measurements were carried out at a sample pressure of 0.010 mbar. Since C₃OS is rather sensitive to the presence of water vapour in the absorption cell, we conditioned the cell for several hours with C₃O₂ which removed all traces of water very effectively.

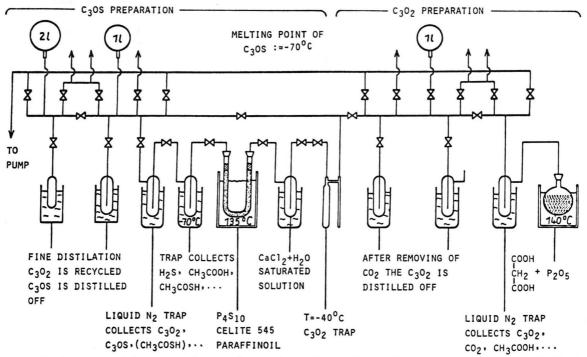


Fig. 1. Block diagram of the vacuum line and the experimental set-up for the chemical synthesis of O=C=C=C=S and its purification via distillation.

III. Spectral Data and Strategy for their Analysis

An overview of the $J = 11 \leftarrow 10$ rotational transition is shown in Figure 2. The vibrational satellite lines arising from the excitation of the v_7 fundamental vibration can be followed very easily up to $v_7 = 9$ in Figure 2. The other, not specifically labeled, satellite lines arise from molecules in the other two bending states, v_5 and v_6 , and their combination states with ν_7 , $2\nu_7$, etc. In Figs. 3-7 expanded segments of the same spectrum are shown with the assignment of the lines. As can be clearly seen from the figures, the higher the excitation of the v_7 vibration the more satellite lines occur in each group of lines which obviously belong together. These lines arise from substates which are coupled by l-type resonance interactions. The individual substates are distinguished by the vibrational angular momentum quantum number l. The data presented in this paper cover 11 rotational transitions from $J=4 \leftarrow 3$ to $J=14 \leftarrow 13$ which have been measured for $v_7 = 0$ to $v_7 = 7$, with l = 0 to l=6 and for $v_5=1$ and $v_6=1$, with l=1. Due to the small dipole moment of C₃OS of 0.662 Debye for the ground vibrational state the observed line width is about 150 kHz [7].

a) The Energy Matrix of a Linear Molecule

The Hamiltonian adopted for the analysis of the experimental data is based on the formalism developed by Nielsen [9, 10] and Amat and Nielsen [11]. This theory rests on three assumptions:

- 1) The equilibrium structure of the molecule is linear.
- The amplitudes of the normal modes of vibration are small.
- 3) The potential function associated with each of the three bending modes of the C₃OS molecule may be represented by a two-dimensional isotropic harmonic oscillator with slight anharmonicity.

The unperturbed rovibrational energy E_{RV} which results when some of the larger effects of anharmonicity are included in the Hamiltonian may be approximately expressed by the diagonal matrix element [12]. The analytical expression of the diagonal matrix element is reduced for this discussion to the case in which only one bending mode, for example ν_7 , is excited:

$$\langle v_t, l_t | \hat{H} | v_t, l_t \rangle = (E_{\text{Vib}} + E_{\text{Rot}}),$$
 (1)

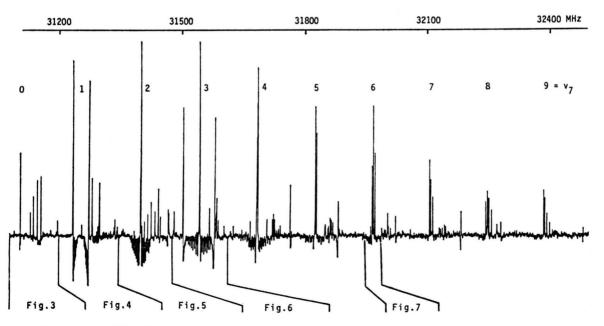


Fig. 2. Survey scan of the $J=11 \leftarrow 10$ rotational transition as observed with a hp MRR 8460 spectrometer, showing the vibrational satellite lines dominated by those arising from energy levels of the vibrational manifold of the lowest lying bending mode ν_7 .

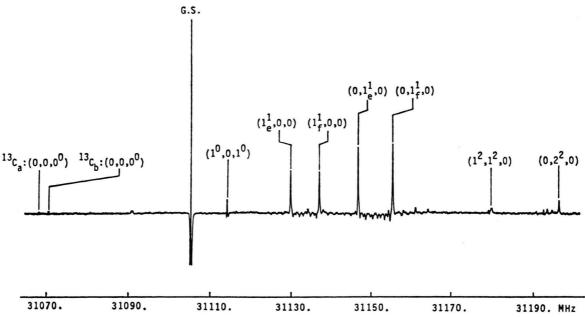


Fig. 3. Expanded scan of a portion of the spectral region shown in Figure 2. The ground state line and vibrational satellites are indicated. The *l*-type doublets of v_5 and v_6 can be clearly recognized. $^{13}C_a$ is $1^{-13}C$ and $^{13}C_b$ is $2^{-13}C$ of O=C=C=C=S.

where

$$\begin{split} E_{\text{Vib}} &= E_0^* + \omega_t(v_t + 1) \\ &+ x_{tt}(v_t + 1)^2 + x_{l_t l_t} l_t^2, \\ E_{\text{Rot}} &= [B_v + \gamma_{l_t l_t} l_t^2] [J(J+1) - l^2] \\ &- D_v [J(J+1) - l^2]^2 \\ &+ H_v [J(J+1) - l^2]^3 \end{split} \tag{3}$$

with

$$B_v = B_e^* - \alpha_t(v_t + 1) + \gamma_{tt}(v_t + 1)^2,$$
 (4)

$$D_{\mathbf{v}} = D_{\mathbf{e}}^* + \beta_t(v_t + 1), \tag{5}$$

where

 E_0 * the zero point energy of the other normal modes:

 ω_t the harmonic frequency for the *t*-th normal mode;

 x_{tt} anharmonicity constant for v_t in the expansion of the vibrational energy;

 $x_{l_t l_t}$ anharmonicity constant which describes the contribution of the vibrational angular momentum l_t to the vibrational energy;

 $B_{\rm e}^*$, $D_{\rm e}^*$ effective equilibrium spectroscopic constants which include the zero point vibration contributions of the other normal modes;

 B_v rotational constant for a given vibrational state;

 α_t , β_t , γ_{tt} rovibrational interaction constants for the *t*-th normal mode;

 $\gamma_{l_t l_t}$ rovibrational interaction constant;

 D_v, H_v centrifugal stretching constants;

sum of the vibrational angular momentum quantum numbers which in this case is $l=l_t$.

All the spectroscopic constants appearing in the expression for the rotational energy (3) and the expansion of the constants B_v (4) and D_v (5) as well as the anharmonicity constant $x_{l,l}$ may be obtained from the rotational spectrum. For a given J, the energy difference between l sublevels may be seen from (2) and (3) to be given by

$$\begin{split} g_{l_t l_t} \, l^2 &= (x_{l_t \, l_t} - B_v - \gamma_{l_t l_t} l_t^2 - D_v \, l^2 \\ &\quad - H_v \, l^4) \, l^2 \\ &\cong (x_{l_t l_t} - B_v) \, l^2. \end{split} \tag{6}$$

Using the second order transformed Hamiltonian [11], only states of different l but with the same v and J are connected by off-diagonal matrix elements of which the most important matrix elements

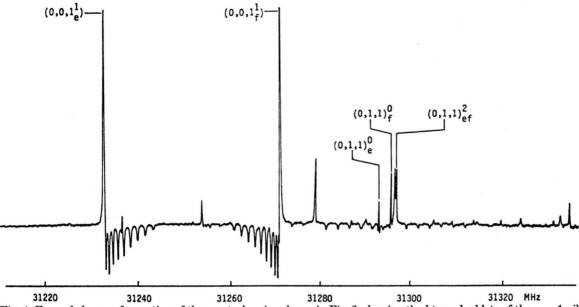


Fig. 4. Expanded scan of a portion of the spectral region shown in Fig. 2, showing the l-type doublets of the $v_7 = 1$ vibrational state.

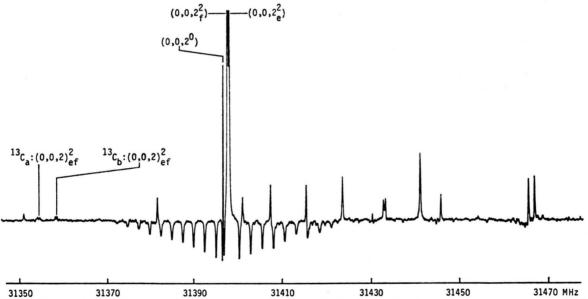


Fig. 5. Expanded scan of a portion of the spectral region shown in Fig. 2, showing the $v_7 = 2$ l-type resonance multiplet. $^{13}\text{C}_a$ is $1 - ^{13}\text{C}$ and $^{13}\text{C}_b$ is $2 - ^{13}\text{C}$ of 0 = C = C = C = S.

are of the type*

$$\langle v_t, l_t | \hat{H}_2' | v_t, l_t \pm 2 \rangle$$

$$= \frac{q_t}{4} \{ (v_t \mp l_t) (v_t \pm l_t + 2) \cdot [J(J+1) - l(l \pm 1)] \cdot [J(J+1) - (l \pm 1)(l \pm 2)] \}^{1/2}, \quad (7)$$

and has been discussed in detail in [12]. This type of matrix element contributes to every excited state $v_t > 0$. We can neglect the higher order interactions expressed by higher order matrix elements

* The phase factor taken here is different from that taken in Ref. [10, 19] and results in a positive off-diagonal matrix element, and thus a positive definition of q_t .

as discussed in Ref. [12] in the case of the present data, though we did find it necessary to include the J-dependence of q_7 :

$$q_7 = q_7^{(0)} - q_7^{(1)} J(J+1).$$
 (8)

From a theoretical point of view the discussion of the energy levels belonging to these vibrational states is straightforward. However, in practice the size of the energy matrix and the number of matrix elements becomes large with increasing v. For that reason we developed a fitting program which can handle in a systematic way the l-type resonance in any vibrational state for a single bending mode in a linear molecule, limited only by the array size in the program.

The analysis was approached by exploiting the analogy between the energy matrix of a linear molecule in various excited states of a single bending mode and that of an asymmetric rotor molecule in a single vibrational state. The angular momentum quantum number l in the former case and $K = K_a$ in the latter case are equivalent [10, 13]; the overall parity + or - of the rovibrational states has the same meaning in both types of molecules [14] and the correlation between the associated energy levels has been discussed frequently [4]. The offdiagonal matrix elements are only $\Delta l = \pm 2$ in the former case, and only $\Delta K = +2$ in the latter case (at least in the A reduction of the Hamiltonian [15]), and even have the same J and K dependence [16, 17]. Therefore, the method and notation used in the analysis of an asymmetric rotor spectrum was applied in the organization of a fitting program for the analysis of the present C₃OS data.

There is a separate energy matrix for each vibrational quantum number v and for each rotational quantum number J. Each block of the energy matrix is of size $(v+1)\times(v+1)$ and tridiagonal. The energy matrix for even v is called $|\hat{E}|$ and is composed only of even-l basis functions, just as the even-K submatrix of an asymmetric rotor [17]. Similarly, the energy matrix for the odd vibrational states is composed of only odd-l basis functions and is called $|\hat{O}|$, in analogy to the odd-K submatrix of an asymmetric rotor [17].

b) The Wang Transformation and the Symmetry of Rovibrational Energy Levels

Since the energy matrices thus obtained have the same forms as those of an asymmetric rotor, they can be further factorized into two submatrices by applying a Wang transformation of the basis functions:

$$\Psi^{\pm}(v,J,|l|)
= [|v,J,|l|\rangle \pm |v,J,-|l|\rangle]/\sqrt{2}$$
(9)

for $l \neq 0$, and

$$\Psi^{+}(v,J,0) = |v,J,0\rangle, \tag{10}$$

where $|v, J, \pm|l|\rangle$ is the basis wavefunction of a linear molecule, and $\Psi^{\pm}(v, J, |l|)$ is the transformed wavefunction. The transformed matrices after rearranging the rows and columns are the submatrices $|\hat{E}|^+$, $|\hat{E}|^-$, $|\hat{O}|^+$, $|\hat{O}|^-$. Since these submatrices are the same as those of an asymmetric rotor the reader is referred to the detailed discussion by Gordy and Cook [17] where the explicit form of the matrices is given.

In our choice of the phase factor of the basis wavefunction, the symmetry of the wavefunction $\Psi^{\pm}(v, J, |l|)$ is obtained* as listed in Table 1, where the correspondence among symmetry designations for wavefunctions, energy submatrices and energy levels is summarized. When the l-type coupling constant q_t is positive, as is generally the case for linear molecules, an eigenvalue of the $|\hat{O}|$ + matrix, an f state [18], corresponds to the upper component (U) of an l-type doublet. An eigenvalue of the $|\hat{O}|$ - matrix, an e state, corresponds to the lower component (L). The opposite is the case if the constant q_t is negative. The use of the e and e notation included in Table 1 has been recommended by Brown et al. [18].

On the other hand, the symmetry of the levels of even v, the eigenstates of the $|\hat{E}|$ = matrices, cannot be correlated simply to upper and lower components of the l-type resonance doublets. As discussed in detail in Ref. [12], the symmetry has to be identified by finding a relation between the level in question and an l=0 level, which is an e state for a $^{1}\Sigma^{+}$ vibronic state and an f state for a $^{1}\Sigma^{-}$ vibronic state. Such a procedure of symmetry identification has to be used also for an odd v state if a rigorously unambiguous classification of the level is needed, rather than the above correspondence based on assuming the sign of q. For an l=1 state the unambiguous assignment is possible by observing an $l=0 \rightarrow l=1$ transition, or by observing an accidental resonance between an l=0 and an l=1 state.

| $\begin{array}{cc} \text{Sub-} & l \\ \text{matrix} \end{array}$ | | γ | Parity | | EPI Group Symmetry ^a | | State Index ^b | $l	ext{-Doublet Comp.}^{\mathrm{c}}$ | |
|--|------------|---|---------------------|------------------------|--|------------------------|-----------------------------|--|---|
| | | | \overline{J} even | $J \operatorname{odd}$ | \overline{J} even | $J \operatorname{odd}$ | | | |
| $ \widehat{E} ^+$ | even | + | + | _ | Σ^+ | Σ- | e | $g_{l_t l_t} > 0$ U | $g_{l_t l_t} < 0$ L |
| $ \widehat{E} ^-$ | even | - | _ | + | Σ^{-} | $arSigma^+$ | f | \mathbf{L} | \mathbf{U} |
| Ô + Ô - | odd odd | + | _ + | + | $rac{\mathcal{\Sigma}^{-}}{\mathcal{\Sigma}^{+}}$ | $\Sigma^+ \ \Sigma^-$ | f e | $egin{aligned} q_{m{t}} > 0 \ \mathrm{U} \ \mathrm{L} \end{aligned}$ | $egin{aligned} q_{t} < 0 \ \mathrm{L} \ \mathrm{U} \end{aligned}$ |

Table 1. Symmetry Classification of the Wavefunction $\Psi^{\nu}(v,J,l)$ and Submatrices of Excited States of a Single Bending Mode of a Linear Molecule in a ${}^{1}\Sigma^{+}$ Electronic State.

^a Notation recommended by Bunker and Papousek [14].
^b Notation recommended by Brown et al. [18].

^c Lower and upper component of the *l*-type doubling are indicated by L and U, respectively rather than a and b as suggested in Ref. [18], because a and b are often used to indicate the symmetry for C₂.

IV. Assigment and Analysis of C₃OS Data

a) Assignment

For an odd v state, the l=1 level splits into two well-separated substates of symmetry species e and f due to the off-diagonal matrix element given in (7). This correction to the energy is the largest among the various vibration-rotation corrections, since the substates are otherwise degenerate and the correction is a first-order correction. Therefore, we can assign the levels with the aid of the relations given in Table 1 for $|\hat{O}|$ submatrices assuming q_7 > 0. For even v states, the l=2 substate splits into e and f substates, which can be assigned as follows: When $g_{l,l}$ is positive the l=2 level is higher in energy than the l=0 level. Thus the e component of the l=2 state is pushed up by the l-type resonance interaction. Similarly, when the constant $g_{l_{t}l_{t}}$ is negative, the e energy level is pushed down. Thus the e level is higher than the f level for $g_{l,l} > 0$, and vice versa, as listed in Table 1. The corrections discussed for the |l|=1 or |l|=2 states propagate into higher |l| eigenvalues, as in an asymmetric rotor. The relations given in Table 1 are valid except when accidental perturbations are present. It is not necessary in the present data to distinguish e and f in the $|l| \ge 3$ substates, because the energy separation is very small and cannot be detected under Doppler resolution in the frequency range covered.

Since the sign of the constant $g_{l,l}$ was not known, we had to extract this information from the effects of the l-type resonance in order to identify the e and f components in the observed spectrum. The δ -plot [19] is helpful in this respect. The quantity δ is defined for a pure rotational transition in a given

(v, l) state as

$$\delta = \frac{v}{2(J+1)} - B_{v,l} + 2D_0(J+1)^2,$$
 (11)

where $B_{v,l}$ is the effective rotational constant for the excited state obtained by fitting all the data for a given v, l component to the expression

$$v = 2(J+1) B_{v,l} - 4(J+1)^3 D_{v,l},$$
 (12)

and D_0 is the centrifugal distortion constant in the ground vibrational state. This quantity represents the deviation of the effective centrifugal distortion contribution in an excited vibrational state from that in the unperturbed case, represented by the ground vibrational state. An example of the δ plot for the v = 6 state of C₃OS is shown in Figure 8. The l=0 substate and the upper component of the l=2state show large, symmetrical shifts relative to the vertical which indicate that these levels are coupled by the *l*-type resonance interaction. Since only levels of the same rovibrational symmetry can be coupled, we can conclude that the l=0 and $l=2_U$ levels are of the same symmetry and can thus be labeled e. In addition we may conclude that $g_{l,l}$ is positive for this particular system of interacting

b) Least Squares Analysis of the Measured Lines

The measured vibrational satellite lines collected in Table 2 are R-branch transitions for which $\Delta J = +1$ and $\Delta l = 0$. The structure of the non-linear least squares fitting program is modelled after our asymmetric rotor programs. The quantum numbers and symmetry assignments are used to select the appropriate energy levels from the submatrix eigenvalues. The program sets up the Jacobian matrix, which is the derivative of the energy with respect

Table 2. Observed Rotational Frequencies of C₃OS in MHz and Deviations from the Calculated Frequencies for the Vibrational States Indicated by (v_5^l, v_6^l, v_7^l) .

| $J+1\leftarrow\!\!J$ | (0, 0, 0) | | $(1_e^1, 0, 0)$ | | $(1_{\rm f}^1,0,0)$ | | $(0,1_e^1,0)$ | |
|---|--|--|---|--|--|--|--|---|
| | $ u_{ m obs}$ | O-C | $v_{ m obs}$ | О-С | $v_{ m obs}$ | O-C | $v_{ m obs}$ | O-C |
| 4 ← 3 | 11311.284 | - 0.004 | 11320.204 | 0.013 | 11322.759 | - 0.017 | 11326.258 | 0.009 |
| $5 \leftarrow 4$ | 14139.110 | 0.008 | 14150.232 | 0.001 | 14153.461 | -0.001 | 14157.819 | 0.016 |
| $6 \leftarrow 5$ | 16966.913 | 0.002 | 16980.271 | 0.007 | 16984.133 | -0.009 | 16989.366 | 0.013 |
| $7 \leftarrow 6$ | | -0.010 | 19810.302 | 0.011 | 19814.809 | -0.006 | 19820.889 | -0.007 |
| $8 \leftarrow 7$ | 22622.513 | 0.004 | 22640.300 | -0.010 | 22645.479 | -0.001 | 22652.428 | -0.004 |
| $9 \leftarrow 8$ | | -0.003 | 25470.308 | -0.012 | 25476.135 | -0.001 | 25483.953 | -0.007 |
| 10 ← 9 | 28278.074 | 0.002 | 28300.316 | -0.003 | 28306.793 | 0.011 | 28315.472 | -0.007 |
| $11 \leftarrow 10$ | 31105.845 | 0.006 | 31130.306 | -0.001 | 31137.425 | 0.009 | 31146.995 | 0.008 |
| $12 \leftarrow 11$ | 33933.587 | -0.007 0.005 | 33960.295 36790.239 | $-0.012 \\ -0.006$ | 33968.049 | -0.011 -0.007 | $33978.469 \\ 36809.979$ | -0.016 0.009 |
| $13 \leftarrow 12$ $14 \leftarrow 13$ | 36761.341 39589.063 | -0.002 | 39620.191 | -0.000 -0.001 | 36798.639 39629.234 | -0.007 -0.005 | 39641.442 | 0.003 |
| ————————————————————————————————————— | 39369.003 | - 0.002 | 39020.191 | - 0.001 | | - 0.005 | 39041.442 | 0.001 |
| $\overline{J+1\leftarrow J}$ | $(0,1_{t}^{1},0)$ | | $(0,0,1^1_e)$ | | $(0,0,1^1_t)$ | | $(0, 0, 2^0)$ | |
| | $v_{ m obs}$ | O-C | $v_{ m obs}$ | O-C | $v_{ m obs}$ | O-C | $v_{ m obs}$ | О-С |
| 4 ← 3 | | - 0.015 | 11357.422 | 0.006 | 11371.413 | - 0.006 | 11417.134 | 0.011 |
| $5 \leftarrow 4$ | 14161.636 | -0.010 | 14196.769 | 0.008 | 14214.271 | 0.007 | 14271.378 | -0.001 |
| $6 \leftarrow 5$ | 16993.953 | -0.011 | 17036.102 | 0.003 | 17057.094 | -0.008 | 17125.628 | 0.010 |
| $7 \leftarrow 6$ | 19826.280 | 0.005 | 19875.427 | -0.002 | 19899.942 | 0.010 | 19979.836 | -0.001 |
| $8 \leftarrow 7$ | 22658.583 | 0.003 | 22714.746 | -0.004 | 22742.751 | -0.002 | 22834.039 | 0.006 |
| $9 \leftarrow 8$ | 25490.876 | 0.000 | 25554.050 | -0.011 | $\begin{array}{c} 25585.553 \\ 28428.369 \end{array}$ | -0.010 | 25688.212 | 0.010 |
| $ \begin{array}{c} 10 \leftarrow 9 \\ 11 \leftarrow 10 \end{array} $ | 28323.175 31155.440 | -0.011 -0.001 | $28393.371 \\ 31232.632$ | -0.011 -0.014 | 31271.148 | $0.009 \\ 0.003$ | $\begin{array}{c} 28542.345 \\ 31396.446 \end{array}$ | -0.004 -0.001 |
| $11 \leftarrow 10$ $12 \leftarrow 11$ | 33987.716 | 0.010 | 34071.925 | -0.014 0.007 | 34113.916 | 0.003 | 34250.513 | -0.001 |
| $12 \leftarrow 11$ $13 \leftarrow 12$ | 36819.959 | -0.001 | 36911.176 | 0.001 | 36956.658 | -0.001 | 37104.539 | -0.003 -0.006 |
| $14 \leftarrow 13$ | 39652.194 | -0.006 | 39750.415 | 0.001 | 39799.410 | 0.005 | 39958.519 | -0.011 |
| $J+1\leftarrow J$ | $(0,0,2^2)$ | | $(0,0,2^2)$ | | $(0,0,3_{\rm e}^{\rm i})$ | | $(0, 0, 3^1)$ | |
| J+1~J | | O-C | | O-C | | O-C | - | 0-C |
| | $v_{ m obs}$ | | vobs | | v _{obs} | | $v_{ m obs}$ | |
| | | 0.010 | | 0.006 | | | 11483.736 | 0.003 |
| $4 \leftarrow 3$ | 11417.354 | -0.012 | 11417.354 | | 11455.436 | 0.001 | | |
| $5 \leftarrow 4$ | 14271.683 | -0.025 | 14271.683 | 0.010 | 14319.279 | 0.003 | 14354.650 | 0.002 |
| $\begin{array}{c} 5 \leftarrow 4 \\ 6 \leftarrow 5 \end{array}$ | 14271.683 17126.015 ^a | -0.025 -0.037 | 14271.683 17126.015 ^a | $0.010 \\ 0.025$ | 14319.279 17183.110 | $0.003 \\ 0.006$ | $\frac{14354.650}{17225.541}$ | -0.002 -0.009 |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 $ | 14271.683 17126.015 ^a 19980.340 ^a | -0.025 -0.037 -0.056 | 14271.683 17126.015 ^a 19980.340 ^a | $0.010 \\ 0.025 \\ 0.042$ | 14319.279 17183.110 20046.915 | $0.003 \\ 0.006 \\ -0.003$ | $\begin{array}{c} 14354.650 \\ 17225.541 \\ 20096.435 \end{array}$ | -0.002 -0.009 -0.002 |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 $ | 14271.683 17126.015 ^a 19980.340 ^a 22834.734 | $\begin{array}{c} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \end{array}$ | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 | 0.010 0.025 0.042 -0.001 | 14319.279 17183.110 20046.915 22910.723 | $0.003 \\ 0.006 \\ -0.003 \\ 0.007$ | 14354.650 17225.541 20096.435 22967.307 | $\begin{array}{c} 0.002 \\ -0.009 \\ -0.002 \\ 0.001 \end{array}$ |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 $ | 14271.683 17126.015 ^a 19980.340 ^a 22834.734 25689.055 | $\begin{array}{c} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \end{array}$ | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 25688.907 | 0.010 0.025 0.042 -0.001 0.028 | 14319.279 17183.110 20046.915 22910.723 25774.485 | 0.003 0.006 -0.003 0.007 -0.009 | 14354.650 17225.541 20096.435 22967.307 25838.149 | $\begin{array}{c} 0.002 \\ -0.009 \\ -0.002 \\ 0.001 \\ -0.006 \end{array}$ |
| $ 5 \leftarrow 4 $ $ 6 \leftarrow 5 $ $ 7 \leftarrow 6 $ $ 8 \leftarrow 7 $ $ 9 \leftarrow 8 $ $ 10 \leftarrow 9 $ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 | $\begin{array}{l} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \\ -0.010 \end{array}$ | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 25688.907 28543.157 | 0.010 0.025 0.042 -0.001 0.028 0.008 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 | $\begin{array}{c} 0.003 \\ 0.006 \\ -0.003 \\ 0.007 \\ -0.009 \\ -0.015 \end{array}$ | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 |
| $ 5 \leftarrow 4 $ $ 6 \leftarrow 5 $ $ 7 \leftarrow 6 $ $ 8 \leftarrow 7 $ $ 9 \leftarrow 8 $ $ 10 \leftarrow 9 $ $ 11 \leftarrow 10 $ | 14271.683 17126.015 ^a 19980.340 ^a 22834.734 25689.055 28543.429 31397.788 | $\begin{array}{c} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \\ -0.010 \\ 0.000 \end{array}$ | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 | $\begin{array}{c} 0.010 \\ 0.025 \\ 0.042 \\ -0.001 \\ 0.028 \\ 0.008 \\ 0.001 \end{array}$ | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 | $\begin{array}{c} 0.003 \\ 0.006 \\ -0.003 \\ 0.007 \\ -0.009 \\ -0.015 \\ 0.009 \end{array}$ | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 |
| $5 \leftarrow 4$ $6 \leftarrow 5$ $7 \leftarrow 6$ $8 \leftarrow 7$ $9 \leftarrow 8$ $10 \leftarrow 9$ $11 \leftarrow 10$ $12 \leftarrow 11$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 | $\begin{array}{l} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \\ -0.010 \end{array}$ | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 | $\begin{array}{c} 0.010 \\ 0.025 \\ 0.042 \\ -0.001 \\ 0.028 \\ 0.008 \\ 0.001 \\ 0.008 \end{array}$ | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 | $\begin{array}{c} 0.003 \\ 0.006 \\ -0.003 \\ 0.007 \\ -0.009 \\ -0.015 \end{array}$ | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 |
| $ 5 \leftarrow 4 $ $ 6 \leftarrow 5 $ $ 7 \leftarrow 6 $ $ 8 \leftarrow 7 $ $ 9 \leftarrow 8 $ $ 10 \leftarrow 9 $ $ 11 \leftarrow 10 $ | 14271.683 17126.015 ^a 19980.340 ^a 22834.734 25689.055 28543.429 31397.788 | $\begin{array}{c} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \\ -0.010 \\ 0.000 \\ -0.007 \end{array}$ | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 | $\begin{array}{c} 0.010 \\ 0.025 \\ 0.042 \\ -0.001 \\ 0.028 \\ 0.008 \\ 0.001 \end{array}$ | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 |
| $5 \leftarrow 4$ $6 \leftarrow 5$ $7 \leftarrow 6$ $8 \leftarrow 7$ $9 \leftarrow 8$ $10 \leftarrow 9$ $11 \leftarrow 10$ $12 \leftarrow 11$ $13 \leftarrow 12$ $14 \leftarrow 13$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 | $\begin{array}{c} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \\ -0.010 \\ -0.000 \\ -0.007 \\ -0.000 \end{array}$ | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 | 0.010 0.025 0.042 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 |
| $5 \leftarrow 4$ $6 \leftarrow 5$ $7 \leftarrow 6$ $8 \leftarrow 7$ $9 \leftarrow 8$ $10 \leftarrow 9$ $11 \leftarrow 10$ $12 \leftarrow 11$ $13 \leftarrow 12$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 | 0.010 0.025 0.042 0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 \\ 10 \leftarrow 9 \\ 11 \leftarrow 10 \\ 12 \leftarrow 11 \\ 13 \leftarrow 12 \\ 14 \leftarrow 13 $ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 | $\begin{array}{c} -0.025 \\ -0.037 \\ -0.056 \\ -0.008 \\ -0.034 \\ -0.010 \\ -0.000 \\ -0.007 \\ -0.000 \end{array}$ | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 | 0.010 0.025 0.042 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 \\ 10 \leftarrow 9 \\ 11 \leftarrow 10 \\ 12 \leftarrow 11 \\ 13 \leftarrow 12 \\ 14 \leftarrow 13 $ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3*et) **pobs | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4°) | 0.010 0.025 0.042 0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4°) v _{obs} | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4 ² _t) v _{obs} | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 \\ 10 \leftarrow 9 \\ 11 \leftarrow 10 \\ 12 \leftarrow 11 \\ 13 \leftarrow 12 \\ 14 \leftarrow 13 $ $ \hline J+1 \leftarrow J $ $ 4 \leftarrow 3 \\ 5 \leftarrow 4$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3ar) vobs | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4°) vobs 11521.467 14401.768 | 0.010 0.025 0.042 -0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4°) vobs 11521.785° 14402.241° | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 O-C | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4*) | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 O-C |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 \\ 10 \leftarrow 9 \\ 11 \leftarrow 10 \\ 12 \leftarrow 11 \\ 13 \leftarrow 12 \\ 14 \leftarrow 13 $ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3ar) 2008 2 | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 - 0.008 | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4°) vobs 11521.467 14401.768 17282.036 | 0.010 0.025 0.042 -0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4°) | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 O-C - 0.037 - 0.059 - 0.094 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4 ² _t) | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 O-C 0.021 0.057 0.109 |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 \\ 10 \leftarrow 9 \\ 11 \leftarrow 10 \\ 12 \leftarrow 11 \\ 13 \leftarrow 12 \\ 14 \leftarrow 13 $ $ \hline $ $ J+1 \leftarrow J $ $ 4 \leftarrow 3 \\ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6$ | 14271.683 17126.015 ^a 19980.340 ^a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3 ^a _{et}) **pobs** 11470.067 14337.577 17205.060 20072.555 | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 O-C 0.008 0.011 - 0.008 - 0.008 - 0.008 | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4°) vobs 11521.467 14401.768 17282.036 20162.236 | 0.010 0.025 0.042 - 0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4*) **pobs** 11521.785** 14402.241** 17282.700** 20163.289 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 O-C | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4 ² ₁) | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 O-C |
| $ 5 \leftarrow 4 \\ 6 \leftarrow 5 \\ 7 \leftarrow 6 \\ 8 \leftarrow 7 \\ 9 \leftarrow 8 \\ 10 \leftarrow 9 \\ 11 \leftarrow 10 \\ 12 \leftarrow 11 \\ 13 \leftarrow 12 \\ 14 \leftarrow 13 $ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3a) vobs 11470.067 14337.577 17205.060 20072.555 22940.053 | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 - 0.008 0.011 - 0.008 0.011 - 0.008 0.002 | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4°) | 0.010 0.025 0.042 - 0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4*) vobs 11521.785* 14402.241* 17282.700* 20163.289 23043.821 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 O-C - 0.037 - 0.059 - 0.094 - 0.018 - 0.021 | $14354.650 \\ 17225.541 \\ 20096.435 \\ 22967.307 \\ 25838.149 \\ 28708.988 \\ 31579.787 \\ 34450.565 \\ 37321.290$ $(0, 0, 4^2)$ $0 \\ \nu_{obs}$ $11521.785^a \\ 14402.241^a \\ 17282.700^a \\ 20162.999 \\ 23043.367$ | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 O-C 0.021 0.057 0.109 0.017 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3a) vobs 11470.067 14337.577 17205.060 20072.555 22940.053 25807.532 | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 - 0.008 0.011 - 0.008 - 0.008 0.011 - 0.008 - 0.008 - 0.008 | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4 ^o) vobs 11521.467 14401.768 17282.036 20162.236 23042.408 25922.503 | 0.010 0.025 0.042 - 0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4°) **pobs** 11521.785°* 14402.241°* 17282.700°* 20163.289 23043.821 25924.373 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 O-C - 0.037 - 0.059 - 0.094 - 0.018 - 0.021 - 0.028 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4*) | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 0-C 0.021 0.057 0.109 0.017 0.013 0.027 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3*et) ***pobs** 11470.067 14337.577 17205.060 20072.555 22940.053 25807.532 28674.998 | - 0.025 - 0.037 - 0.056 - 0.008 - 0.034 - 0.010 0.000 - 0.007 0.000 0.008 - 0.008 0.011 - 0.008 0.011 - 0.008 0.002 | 14271.683 17126.015a 19980.340a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4°) | 0.010 0.025 0.042 -0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4°) ***/pobs** 11521.785°** 14402.241°** 17282.700°** 20163.289 23043.821 25924.373 28804.985 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 - 0.003 - 0.059 - 0.094 - 0.018 - 0.021 - 0.028 - 0.003 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4*) | 0.002 - 0.009 - 0.001 - 0.006 0.007 0.004 0.009 - 0.010 O-C 0.021 0.057 0.109 0.017 0.013 0.027 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 14271.683 17126.015a 19980.340a 22834.734 25689.055 28543.429 31397.788 34252.134 37106.495 39960.859 (0, 0, 3a) vobs 11470.067 14337.577 17205.060 20072.555 22940.053 25807.532 | | 14271.683 17126.015 ^a 19980.340 ^a 22834.594 25688.907 28543.157 31397.403 34251.647 37105.860 39960.058 (0, 0, 4 ^o) vobs 11521.467 14401.768 17282.036 20162.236 23042.408 25922.503 | 0.010 0.025 0.042 - 0.001 0.028 0.008 0.001 0.008 0.003 0.005 | 14319.279 17183.110 20046.915 22910.723 25774.485 28638.235 31501.991 34365.695 37229.363 (0, 0, 4°) **pobs** 11521.785°* 14402.241°* 17282.700°* 20163.289 23043.821 25924.373 | 0.003 0.006 - 0.003 0.007 - 0.009 - 0.015 0.009 0.007 - 0.003 O-C - 0.037 - 0.059 - 0.094 - 0.018 - 0.021 - 0.028 | 14354.650 17225.541 20096.435 22967.307 25838.149 28708.988 31579.787 34450.565 37321.290 (0, 0, 4*) | 0.002 - 0.009 - 0.002 0.001 - 0.006 0.007 0.004 0.009 - 0.010 O-C 0.021 0.057 0.109 0.017 0.013 0.027 0.004 |

Table 2 continued.

| $J+1\leftarrow J$ | $(0,0,4_{ m ef}^4)$ | | $(0, 0, 5_{e}^{1})$ | | | $(0,0,5^1_t)$ | | $(0,0,5_{\rm et}^3)$ | |
|---|--------------------------|--------|----------------------|------------------|-------------------|-------------------------|---------|----------------------|---------|
| | $ u_{\mathrm{obs}} $ | O-C | $v_{ m obs}$ | (|)-C | vobs | 0-C | $v_{ m obs}$ | О-С |
| 4 ← 3 | | | 11551.699 | 9 0 | .012 | 11594.528 | 0.005 | 11573.678 | - 0.009 |
| $5 \leftarrow 4$ | 14403.151 | -0.009 | 14439.593 | 3 0 | .014 | 14493.140 | 0.019 | 14467.090 | -0.011 |
| $6 \leftarrow 5$ | 17283.763 | -0.013 | 17327.463 | 3 0 | .013 | 17391.707 | 0.010 | 17360.491 | -0.020 |
| $7 \leftarrow 6$ | 20164.383 | -0.001 | 20215.302 0. | | .005 | 20290.256 | 0.009 | 20253.902 | -0.013 |
| $8 \leftarrow 7$ | 23044.977 | -0.005 | 23103.11 | 7 0 | .002 | 23188.774 | 0.008 | 23147.304 | -0.007 |
| $9 \leftarrow 8$ | 25925.564 | -0.005 | 25990.899 | -0 | .003 | 26087.248 | -0.002 | 26040.695b | -0.006 |
| $10 \leftarrow 9$ | 28806.149 | 0.006 | 28878.659 | 9 0 | .007 | 28985.698 | 0.003 | 28934.076b | -0.006 |
| $11 \leftarrow 10$ | 31686.708 | 0.005 | 31766.364 | 4 0 | .002 | 31884.095 | -0.001 | 31827.451b | -0.002 |
| $12 \leftarrow 11$ | 34567.252 | 0.005 | 34654.023 - 0.0 | | .005 | 34782.439 | -0.009 | 34720.809b | -0.004 |
| 13 ← 12 ———————————————————————————————————— | 37447.780 | 0.005 | 37541.642 | $\frac{2}{}$ - 0 | .004 | 37680.748 | 0.000 | 37614.168b | 0.006 |
| | $(0, 0, 5_{et}^{5})$ | | (0,0,60) | | | $(0, 0, 6^2)$ | | $(0, 0, 6^2)$ | |
| $J+1\leftarrow J$ | (0, 0, 5 _{et}) | | (0, 0, 0°) | | | (0, 0, 0 _e) | | (0, 0, 0,) | |
| | $v_{ m obs}$ | О-С | $v_{ m obs}$ | (|)-C 1 | obs | о-с | $v_{ m obs}$ | O-C |
| $5 \leftarrow 4$ | | | 14529.957 | | | | - 0.111 | 14530.658a | 0.138 |
| $6 \leftarrow 5$ | 17362.125 | -0.002 | 17435.769 | | | | -0.015 | 17436.609 | 0.031 |
| $7 \leftarrow 6$ | 20255.789 | 0.000 | 20341.499 | | | | -0.008 | 20342.623 | 0.013 |
| $8 \leftarrow 7$ | 23149.438 | -0.001 | 23247.140 | | | | -0.002 | 23248.624 | 0.011 |
| $9 \leftarrow 8$ | 26043.072 | -0.004 | 26152.647 | | | 26156.078 | 0.004 | 26154.592 | 0.009 |
| $10 \leftarrow 9$ | 28936.698 | 0.000 | 29058.022 | | | 29062.577 | 0.013 | 29060.517 | 0.002 |
| $11 \leftarrow 10$ | 31830.306 | 0.003 | 31963.25 | | | 31969.135 | 0.000 | 31966.403 | -0.002 |
| $12 \leftarrow 11$ | 34723.889 | 0.000 | 34868.312 | | | 34875.815 | 0.018 | 34872.239 | -0.009 |
| 13 ← 12 ———————————————————————————————————— | 37617.470 | 0.015 | 37773.177 | 7 -0 | .012 | 37782.586 | 0.031 | 37778.056 | 0.014 |
| | | | | | | | | | |
| $J+1\leftarrow J$ | $(0,0,6_{ef}^{4})$ | | $(0, 0, 6_{ef}^{6})$ | | $J+1\leftarrow J$ | $(0,0,7_{e}^{1})$ | | $(0,0,7^1)$ | |
| | $v_{ m obs}$ | O-C | ν _{obs} | O-C | _ | $v_{ m obs}$ | 0-C | $v_{ m obs}$ | O-C |
| $5 \leftarrow 4$ | 14531.622 | -0.025 | | | $8 \leftarrow 7$ | 23291.801 | 0.001 | 23406.907 | 0.001 |
| $6 \leftarrow 5$ | 17437.932 | -0.029 | | | $9 \leftarrow 8$ | 26203.119 | -0.001 | 26332.586 | -0.001 |
| $7 \leftarrow 6$ | 20344.235 | -0.032 | 20346.761 | 0.018 | | | | | |
| $8 \leftarrow 7$ | 23250.523 | -0.041 | 23253.406 | 0.022 | $J+1\leftarrow J$ | $(0, 0, 7^3_{ef})$ | | $(0, 0, 7_{et}^5)$ | |
| $9 \leftarrow 8$ | 26156.813 | -0.036 | 26160.021 | 0.011 | 5 2.0 | (°, °, ·ef) | | | |
| 10 ← 9 | 29063.080 | -0.042 | 29066.636 | 0.018 | | $v_{ m obs}$ | O-C | $v_{ m obs}$ | O-C |
| 11 ← 10 | 31969.337 | -0.044 | 31973.227 | 0.018 | | | | | |
| $12 \leftarrow 11$ | 34875.560 | -0.066 | 34879.803 | 0.024 | $8 \leftarrow 7$ | 23351.041 | -0.002 | 23353.254 | 0.001 |
| $13 \leftarrow 12$ | 37781.812 | -0.041 | 37786.351 | 0.025 | $9 \leftarrow 8$ | 26269.913 ^t | | 26272.363 | -0.001 |
| | | | / / | | | | | | |

a Lines not used in fit due to unresolved doublet.

to the molecular parameters, via the Hellman-Feynman theorem [20],

$$\frac{\partial \varepsilon}{\partial \lambda} = \int \Psi^* \frac{\partial \hat{H}}{\partial \lambda} \Psi d\tau, \qquad (13)$$

where $\hat{H}\Psi = \varepsilon \Psi$, and λ is one of the molecular parameters.

This fast and compact method has been used with success in our programs for the centrifugal distortion analysis of asymmetric rotor spectra. The matrix diagonalisation section of the program utilizes the Rutishauser-procedure [21] while the linear least squares subroutine and error calculations are standard. The entire procedure is iterated until a convergence is reached.

The adjusted constants for C_3OS in the successive vibrational states of the ν_7 bending mode and the first excited states of ν_5 and ν_6 are given in Table 3. Using these constants the values of the frequencies of the transitions given in Table 2 were calculated.

^b Calculated splitting due to *l*-type resonance interaction 2 to 13 kHz. Therefore, the average value of the calculated line positions was used in determining O-C.

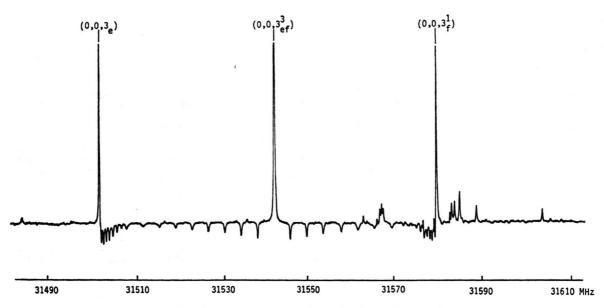


Fig. 6. Expanded region of a portion of the spectral region shown in Fig. 2, showing the $v_7 = 3$ satellite lines. $(0, 0, 3_e)$ should read $(0, 0, 3_e)$.

Table 3. Spectroscopic Constants of C₃OS in Excited Vibrational States.

| (v_5, v_6, v_7) | B_{v_t} / MHz | D_{v_t}/Hz | $q_{v_t^{(0)}}/\mathrm{MHz}$ | $q_{v_t}^{(1)}/\mathrm{Hz}$ | $\gamma_{l_t l_t}/\mathrm{kHz}$ | $x_{l_t l_t}/\mathrm{cm}^{-1}$ | $\sigma/\mathrm{KHz}^{\mathrm{t}}$ |
|-------------------|--------------------------|-----------------------|------------------------------|-----------------------------|---------------------------------|--------------------------------|------------------------------------|
| (0, 0, 0) | 1413.91242 (25) | 43.98 (91) | | | | | 6.1 |
| (0, 0, 1) | 1420.54876 (23) | 54.84 (85) | 1.75041 (47) | 1.90 (171) | 5.30a | | 8.1 |
| (0, 0, 2) | 1427.14471 (40) | 65.95 (116) | 1.75952a | 1.99a | 6.346 (103) | 0.4007(41) | 12.4 |
| (0, 0, 3) | 1433.69383 (27) | 73.19 (87) | 1.76872 (24) | 2.07 (100) | 7.098 (50) | 0.3793(99) | 7.3 |
| (0, 0, 4) | 1440.19162 (33) | 85.75 (124) | 1.77600a | 2.72a | 7.816 (18) | 0.3742(9) | 11.6 |
| (0, 0, 5) | 1446.63449 (22) | 95.26 (82) | 1.78496 (19) | 4.31 (80) | 8.419 (10) | 0.3799(19) | 8.8 |
| (0, 0, 6) | 1453.02344 (61) | 103.23 (241) | 1.79248a | 6.84a | 8.800 (17) | 0.3587(9) | 25.2 |
| (0, 0, 7) | 1459.35934 (46) | 114.69 (311) | 1.79989 (36) | 10.30 (246) | 8.926 (10) | 0.3637(12) | 2.8 |
| (0, 1, 0) | 1415.97451 (28) | 42.63 (102) | 0.38424 (22) | | | | 9.7 |
| (1, 0, 0) | 1415.18691 (26) | 47.41 (96) | 0.32312 (10) | | | | 9.1 |

a Values interpolated or extrapolated and held fixed in fit. b Standard deriation of the fit.

Table 4. Parameters obtained by fitting the Spectroscopic constants given in Table 3 to a power series Expansion in (v+1)

| $B_{7(e)}^*$ a $B_{7(1)} = -\alpha$ $B_{7(2)} = \gamma_{77}$ $B_{7(3)}$ | 7 = = | 407.2298 (48) 6.7018 (43) -0.01984 (109) -0.422 (80) | MHz MHz MHz kHz | $\gamma_{l_7 l_7(e)}^{\bullet} = 2.791 (289) \text{ kHz}$ $\gamma_{l_7 l_7(1)} = 1.411 (112) \text{ kHz}$ $\gamma_{l_7 l_7(2)} = -0.080 (10) \text{ kHz}$ | $D_{7(e)}^{*}$ $D_{7(1)}^{*}$ | $= 34.75 (89) = \beta_7 9.97 (18)$ | Hz Hz |
|---|----------|---|--------------------------|---|---|------------------------------------|----------|
| q _{7(e)} q ₇₍₁₎ q ₇₍₁₎ | = | 1.73483 (147) 0.008234 (269) | MHz MHz | $q_{7(6)}^{(1)*} = -5.06 (105) \text{ Hz}^{a}$ $q_{7(1)}^{(1)} = 2.27 (48) \text{ Hz}$ $q_{7(2)}^{(1)} = -0.364 (47) \text{ Hz}$ | $x_{l_{7}l_{7}(e)}^{ullet} \ x_{l_{7}l_{7}(1)}$ | = 0.4140 (111) $= -0.0069 (19)$ | • |

^a The power series expansion in terms of (v_7+1) may not be physically applicable for $q_7^{(1)}$.

V. Discussion

The adjusted spectroscopic constants of C_3OS collected in Table 3 are representative for a well-behaved linear molecule. The l-type doubling constant is assumed positive for all three bending modes*. The rotational constants B_v and D_v can be expanded according to (4) and (5) and the results of a fit to the (equally weighted) values of $B_{v_{\tau}}$ and $D_{v_{\tau}}$ are given in Table 4. A similar expansion of several of the other molecular parameters as a function of vibrational quantum number v_{τ}

$$P_t = P_{t(e)}^* + P_{t(v)}^*(v_7 + 1) + \dots$$
 (14)

is necessary and the results of the corresponding fits are also given in Table 4. The variation of $q_2^{(1)}$ with v is not well determined and not clearly linear. It should be pointed out that the sign of $x_{l_1 l_2}$ is positive, and $x_{l_1l_1}$ is much larger than B_v for all reported states. This means that the higher l-substates lie at successively higher energy. Only a small decrease of the magnitude of the constant $x_{l_2l_2}$ can be observed with increasing v_7 . In the $v_7 = 6$ state systematic deviations are observed in the least squares fit; the variation of $B_{v,l}$ with l no longer goes as l^2 as could be seen clearly from survey scans for $v_7 > 7$. A trial fit with an l^4 correction to B_v reduced the deviations for $v_7 = 6$ and changes the parameters $\gamma_{l,l}$ and $x_{l,l}$ slightly. However, since this term is determined only for $v_7 = 6$ and no precision measurments have yet been made for $v_7 > 7$ or l>6 this term has not been included in the analysis presented here.

The positive sign of x_{l_1,l_2} and its small dependence on the vibrational quantum number v_7 imply a rather harmonic bending potential function for v_7 . The vibrational energy levels must then be equidistant. A forthcoming publication [7, 22] will report our determination of the vibrational term values of the v_7 vibrational manifold whose fundamental was found to be 82.9 cm⁻¹. Of course, these results are in sharp contrast to the quasilinearity of C_3O_2 [2, 5] whose lowest lying fundamental vibration was found to be 18.2 cm⁻¹ [3].

Long linear carbon chain molecules have rather flexible structures with low — lying, large — amplitude bending states which are well populated at room temperature. Recently Hutchinson, Kroto and Walton [23] have studied cyanobutadiyne, H—CC—CC—CN, and reported the analysis of the

lowest lying vibrational manifold for which $\nu_{11} \cong 75 \text{ cm}^{-1}$.

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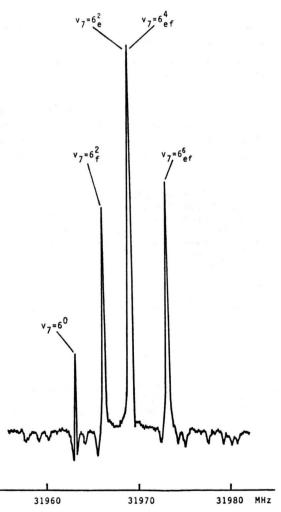


Fig. 7. Expanded region of a portion of the spectral region shown in Fig. 2, showing the $v_7=6$ *l*-type resonance polyade.

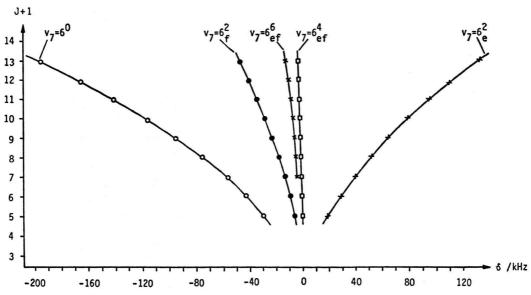


Fig. 8. The quantity δ (see text) is plotted as a function of J+1 showing clearly the interaction between states with like symmetry, 6° and $6_{e^{\circ}}$, which are coupled by the *l*-type resonance.

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